Probability theory rehearsal 2

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If you multiply from the rhs of a matrix, you are working in <u>column space</u>.

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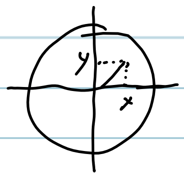
(7) Example of a continuous random variable:



dart board, where all darts are guaranteed to always hit the board. However, the further away you are from the center, the higher your score.

Random variable $X((x,y)) \to |x+y|^2$

(10) Take X with density $f_X(x)$ where $X \rightsquigarrow Y = g(X)$

Y $f_Y(y)$ g is a monotone function

Now since $f_X(x)\,\Delta x \simeq f_Y(\cancel{y})\,\Delta y$ (since g is monotone)

we find $f_Y(y) = f_X(x)\,\frac{\Delta x}{\Delta y}$

(11) An easier method to describe some properties of random variables.

The definition here is just a bit more general than the one you know, viz. $E[X]$ rather than $E[g(X)]$

Note that $\mathbb{I}[X=k] = \begin{cases} 1 & \text{if } X=k \\ 0 & \text{otherwise} \end{cases}$ i.e. a discrete Dirac function.

(13) What's the distribution ~~of~~ with the highest entropy (of a continuous random variable)?

It happens to be the normal distribution (assuming the same mean and variance)!

Kullback-Liebler divergence: if you want to compare two probability mass functions.
An alternative measure is f-divergence.

(14) $\text{Bern.}(p) = \begin{cases} p & \text{for } X=0 \\ 1-p & \text{for } X=1 \end{cases}$

Probability mass function $p_X(x)$ can also be written as

$$p_X(x) = p^{x}(1-p)^{1-x}$$

$\Rightarrow E[X]=p$, ~~Var[X]~~
$Var[X] = p(1-p)$

Binomial (n,p) is obtained by doing n times a Bernoulli(p)

$$\text{Binomial}(n,p) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

"n choose x"

> Notation hint: $X \sim \text{Bern}(p)$ means X is drawn by the Bernoulli distribution

Binomial goes with the central limit theorem, towards a Gaussian (big surprise!)

Poisson counts very rare events...

(15) If you count rare events distributed as Poisson (e.g. counting # red ferraris on the A9), the time you will wait between seeing two ferrari's is distributed as Exponential.

Look up Beta distribution yourself (and know the Normal distribution)!

(19) $Y|X$ means Y given X

$P(y|x)$ is the probability of y if x is given as true.

The subscripts of p used on this slide are there for clarity and mathematical notational correctness. We will leave them out later on, but remember that if we write, e.g.,

$$p(y|x) = \frac{p(x,y)}{p(x)}$$

that these three p's are different things!

p = density, P = probability...

Don't worry! Most of the next ~~...~~ classes are going to be easier, or more entertaining, or... but this "bit" of basis was important.